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Understanding Cross-Sectional Stock Returns: What Really Matters?

Yong Wang

We run a horse race among eight proposed factors and eight proposed conditioning variables for explaining the cross-section of stock returns. The purpose is to better understand which factors, in combination with which conditioning variables, seem robust in explaining cross-sectional data, and to seek an economic interpretation of the specifications that appear most promising. We find that a consumption growth factor, conditioning on lagged business income growth, is the most successful in explaining cross-sectional variation of average quarterly returns in the 25 Fama-French portfolios.

Field of Research: Empirical Finance, Asset Pricing

1. Introduction

Many factor models, with a variety of conditioning variables, have been proposed to explain the cross-section of stock returns. While some are rejected, a number of recent papers have claimed better success. A still open question, however, is what best explains the cross-section variation of average returns, and why? Previous tests are based on different data sets and different methods, making it difficult to compare their performance directly. In this paper, we run a horse race among several factor models and several conditioning variables, comparing their explanatory power for the cross-section of stock returns.

Yong Wang, School of Accounting and Finance, Hong Kong Polytechnic University, Email: afywang@inet.polyu.edu.hk.

Ferson and Harvey (1999), and Hodrick and Zhang (2001) also compare several conditional asset pricing models. While they consider several specific models, we compare all possible models constituted by combining previously proposed factors and conditioning variables. In addition we include several new factors and conditioning variables. A significant difference is in the choice of conditioning variables. The earlier horse races use only one conditioning variable at a time, even for multi-factor models, while we choose different conditioning variables for different factors.

In conditional asset pricing models, conditional variables are used to capture the time variation of risk premium. Conditioning variables are often chosen from those variables that can predict future market returns. Those variables help to capture the variation of aggregate market risk premium, time-varying price of market risk. So they are potential conditioning variables for CAPM, which uses market excess return as factor. However, they are not necessarily good conditioning variables for other factors. Different risk factors have different risk premiums. The variable that can predict aggregate market risk premium may not capture the risk premium variation related to other risk factors. Different risk factors may need different conditioning variables. Nevertheless, all previous tests use only one conditioning variable in one model, no matter how many factors in the model. They all focus on variables that predict market returns, which are potential conditioning variables for market risk factor. In this paper we choose different conditioning variables for different factors, and consider some new conditioning variables.

2. Literature Review

It is well documented in the literature that the CAPM fails to explain the cross-section of stock returns. Roll (1977) points out that the return on the value-weighted portfolio of stocks is unlikely to be an adequate proxy for the return on aggregate wealth, which may be a major cause of the unsatisfactory performance of the CAPM. Many papers have pursued this basic insight. Fama and French (1993) propose a three-factor model with the market excess return, the return on a Small Minus Big size portfolio (SMB), and the return on a High Minus Low book-to-market ratio portfolio (HML) as factors. This three-factor model is statistically successful in explaining the cross-section of stock returns. Jagannathan and Wang (1996) include the return on human capital as a new factor, using labor income growth as a proxy and conditioning on some macroeconomic variable. Heaton and Lucas (2000) introduce proprietary business income growth as a new factor, and find that a linear asset pricing model with proprietary business income has better performance than the similar model that includes only labor income.

The Consumption-based CAPM (CCAPM) developed by Breeden (1979) claims that with complete markets only aggregate consumption risk should be priced. The CCAPM has even less power than the CAPM to explain the cross-section of average asset returns (Campbell, 1996 and Cochrane, 1996). Several researchers have tried to resurrect the CCAPM by decomposing consumption into its component parts. Piazzesi, Schneider and Tuzel (2007) develop a two-factor model in which aggregate consumption risk is transformed into two factors: non-housing consumption growth and expenditure share on non-housing consumption. Lustig and Van Nieuwerburgh (2005) differentiate housing consumption and non-housing consumption as well. In their model with housing collateral they use consumption growth on food and apparel and a rental price growth scaled by housing expenditure share as two factors.

One reason for the failure of static model tests may be that correlation structures and risk premiums are time varying. This implies that time series averages may not reflect the true conditional relationships. Conditional asset pricing models use conditioning variables to capture the time series variation of risk premiums, thereby improving cross-sectional predictability. Several papers have extended the asset pricing literature in this direction. In empirical studies of conditional asset pricing models, conditioning variables are often chosen from those variables that can predict business cycle or forecast the market excess returns. Jagannathan and Wang (1996) use the "default premium", the yield spread between BAA- and AAA-rated bonds, as a proxy for the conditional market risk premium, noting that previous literature shows that it predicts the business cycle and forecasts market excess returns. Dividend yields and term spreads have been shown to have time series predictive ability to forecast stock market returns, so they are also widely used as potential conditioning variables in cross-sectional tests.

Lettau and Ludvigson (2001a) show that fluctuations in the aggregate consumption-wealth ratio are strong predictors of excess returns on the market. Lettau and Ludvigson (2001b) use the log consumption to aggregate wealth ratio (*cay*) as a conditioning variable for both the CAPM and CCAPM. Santos and Veronesi (2006) show that lagged values of labor income to consumption ratio (*s*) can predict stock returns. They also find that both CAPM and CCAPM conditioning on *s* have better performance in the tests of cross-sectional implication than their unconditional counterparts. Lustig and Van Nieuwerburgh (2005) develop a model with housing collateral in which the ratio of house wealth to human wealth (housing collateral ratio, *my*) shifts the conditional distribution of asset prices and consumption growth. They find that a decrease in the house collateral ratio predicts higher returns on stocks, and their conditional model can explain eighty percent of the cross-sectional variation in annual returns on size and book-to-market portfolios. Piazzesi, Tuzel and Schneider (2007) build a model with housing consumption. They find that the non-housing expenditure to total expenditure ratio (α) is both a good forecasting variable for market returns and a good conditional variable for their conditional Consumption-Housing CAPM. In this paper, we reconsider the factors and conditioning variables identified in previous studies. Since they all seem to have some explanatory power for the cross-section of asset returns, a natural first step is to examine the extent to which they are correlated. Surprisingly, we find very little correlation among them, suggesting that they do not all proxy for the same aggregate shocks. To explore which factors, in combination with which conditioning variables, really matter to cross-sectional returns, we run a horse race among eight proposed factors and eight conditioning variables.

3. Methodology

According to first fundamental theorem of asset pricing, in the absence of arbitrage there exists a stochastic discount factor, or pricing kernel, *m*, which prices every asset correctly. That is, the following equation holds:

$$E_t(m_{t+1}R_{t+1}) = 1 \qquad (1)$$

where m_{t+1} is the stochastic discount factor at time t for cash flows arriving at t+1, R_{t+1} is the gross one period return for asset i realized at time t+1.

In this paper, we consider linear factor models, in which stochastic discount factor is a linear function of a constant and a k×1 vector of factors, f_{t+1}. Let model parameters be $[a_t, b_t']$, then the pricing kernel is

$$m_{t+1} = a_t + b_t' f_{t+1} \tag{2}$$

Each linear factor model can be identified by the specific f_{t+1}.

Cochrane (1996) provides one method to incorporate the conditioning information into asset pricing models. He assumes the coefficients in pricing kernel equation (2) are linear functions of conditioning variables. In a one factor model, $a_t = a_0 + a_1 z_t$, $b_t = b_0 + b_1 z_t$, the pricing kernel becomes:

$$m_{t+1} = (a_0 + a_1 z_t) + (b_0 + b_1 z_t) f_{t+1} = a_0 + a_1 z_t + b_0 f_{t+1} + b_1 z_t f_{t+1} \tag{3}$$

Then the conditional model can be rewritten as scaled factor model with constant coefficient:

$$E_t[(a_0 + a_1 z_t + b_0 f_{t+1} + b_1 z_t f_{t+1}) R_{i,t+1}] = 1 \tag{4}$$

In this paper, we estimate and test conditional models in this form.

In previous studies of conditional asset pricing models, conditioning variables are often chosen from those variables that can predict future market returns. Those variables help to capture the variation of aggregate market risk premium, time-varying price of market risk. So they are natural conditioning variables for CAPM, which uses market excess return as factor. However, these variables are not necessarily good conditioning variables for other factors, since they may not reflect the time variation of risk premium related to other factors. In general, different risk factors have different risk premiums. The variable that can predict one factor risk premium may not capture the risk premium variation related to other risk factors. Different risk factors may need different conditioning variables. In this paper, we choose different conditioning variables for different factors. We use Generalized Method of Moments (GMM) to estimate and test the conditional models as in equation (4). For linear factor models, define the pricing error vector:

$$g(b) = E(b'F_tR_t - 1) \tag{5}$$

where b is parameter vector, F_t is factor vector, and R_t is asset return vector. The GMM estimates are formed by choosing b to minimize the weighted sum of pricing errors:

$$\min J = g(b)'W\,g(b) \tag{6}$$

where W is the weighting matrix. There are two weighting matrices widely used in the literature, the optimal weighting matrix and Hansen-Jagannathan weighting matrix. The optimal weighting matrix is proposed by Hansen (1982), $W=S^{-1}$, where S is the covariance matrix of g(b). This weighting matrix is optimal in the sense that the estimated parameters have the smallest asymptotic covariance. Hansen and Jagannathan (1997) propose another weighting matrix, $E[RR']^{-1}$, which is the inverse of the second moments of asset returns. When Hansen-Jagannathan weighting matrix is used, $\sqrt{J}$ measures the minimum distance from the pricing kernel in the model to the set of true pricing kernels, which is often called HJ-distance. While the optimal weighting matrix changes with different models, Hansen-Jagannathan weighting matrix $E[RR']^{-1}$ is invariant across models given the set of test asset returns. Using a common weighting matrix gives a uniform measure of performance across models, so HJ-distance is suitable for model comparisons.

4. Results

We use quarterly data in the U.S. market. For test portfolio returns and factors, the sample period is from 1952Q3 to 2002Q2. Conditioning variables are one quarter lagged, so the sample period is from 1952Q2 to 2002Q1. There are a total of 200 observations for each variable. The test asset returns are the value-weighted returns on the 25 Fama-French portfolios. These portfolios are the intersections of 5 portfolios formed on size and 5 portfolios formed on the ratio of book equity to market equity. The portfolios include all NYSE, AMEX, and NASDAQ stocks. We consider eight factors: Market excess return (R_m), Consumption growth (Δc), Fama-French factors (SMB and HML), Labor income growth (Δy), Proprietary income growth ($\Delta prop$), Scaled rental price change (AΔlogρ), and Non-housing expenditure ratio change (Δlogα). Some of them are from traditional models, such as the market excess return in CAPM. Others, such as housing factors, are proposed in more recent studies.

TABLE 1. Factor Correlation Coefficients

	R_m	Δc	SMB	HML	Δy	Δprop	AΔlgρ	Δlgα
R_m	1.00							
Δc	0.23	1.00						
SMB	0.42	0.13	1.00					
HML	-0.37	-0.02	-0.11	1.00				
Δy	0.11	0.72	0.01	-0.05	1.00			
Δprop	0.21	0.56	0.01	0.06	0.54	1.00		
AΔlgρ	0.01	-0.05	0.03	0.04	0.04	-0.01	1.00	
Δlgα	0.02	0.28	0.17	0.06	0.01	0.15	-0.17	1.00

Table 1 reports the correlation coefficients between these factors. Consumption growth (Δc), labor income growth (Δy) and proprietary income growth (Δprop) are correlated, with correlation coefficients 0.72, 0.56, 0.54. All other correlation coefficients have rather small absolute value. Overall the correlations between these factors are not high. We consider eight variables as potential conditioning variables: Default premium (Default), Term spread (Term), Dividend yield (Dividend), Consumption to wealth ratio (cay), Labor income to consumption ratio (s), Housing collateral ratio (myfa), Non-housing expenditure ratio (α), and Business income growth (big). Seven of them have appeared in previous studies, and we add business income growth to the list. All conditioning variables are lagged one quarter comparing with factors. Table 2 provides the data summary of conditioning variables. The correlations between proposed conditioning variables are moderate. The highest correlation coefficient is only 0.58. Most of them are just around 0.1 and 0.2.

TABLE 2. Conditioning Variable Correlation Coefficients

	Default	Term	Dividend	cay	s	myfa	α	big
Default	1.00							
Term	0.15	1.00						
Dividend	0.46	0.09	1.00					

cay	0.10	0.14	0.38	1.00				
s	0.22	-0.11	0.47	-0.21	1.00			
myfa	0.41	0.00	0.58	0.21	-0.15	1.00		
α	-0.10	-0.05	0.15	-0.10	-0.09	0.29	1.00	
big	0.07	-0.15	0.01	0.03	0.05	-0.04	0.00	1.00

Most of factors and conditioning variables have low correlations with each other, suggesting that they may represent different risks that should be priced. All of these factors and conditioning variables have been found to be important in previous studies. It is natural to ask if some factors are more important than others and if some conditioning variables are better than others. Since most previous tests are based on different data sets and different methods, it is difficult to compare their performances. We run a horse race among these proposed factors and proposed conditioning variables, using same data sets and same method to compare their performances in explaining the cross-section of stock returns.

TABLE 3. Compare Conditioning Variables for One Factor

Factor	Conditioning Variable	HJ-distance	T·J	p-value
Δc	big	0.40	31.30	0.78
Δc	Default	0.53	56.90	0.02
Δc	s	0.56	61.60	0.00
Δc	Term	0.56	62.10	0.00
Δc	cay	0.56	62.10	0.00
Δc	Dividend	0.56	62.20	0.00
Δc	myfa	0.56	62.40	0.00
Δc	α	0.56	63.70	0.00

First we compare all possible one factor conditional models. For each factor, we try eight different conditioning variables, and then choose the one that has smallest pricing error. Table 3 gives an example of comparing performance of one factor conditioning on eight different conditioning variables. The results are ranked by HJ-distance. For consumption risk factor Δc, business income growth (*big*) is the best conditioning variable, with the smallest HJ-distance. We repeat the same procedure for every factor, choose the best conditioning variable for each, and report the results in Table 4. The results are also ranked by HJ-distance. According to Jagannathan and Wang (1996), the asymptotic distribution of T·J is weighted sum of 21 i.i.d. random variables of $\chi^2(1)$ distribution. The p-value is estimated by simulation. Compared with other scaled factors, Δc conditioning on *big* has the best performance. The conditional CCAPM with the best conditioning variable *big* can not be rejected, and the pricing error is not significantly different from zero. The market excess return factor has the worst performance in terms of highest pricing errors. Term spread is the best conditioning variable for market risk factor. Even with the best conditioning variable, the conditional CAPM still can be rejected under 10% significance level.

TABLE 4. One-Factor Models with the Best Conditioning Variable

Factor	Conditioning Variable	HJ-distance	T·J	p-value
Δc	big	0.40	31.30	0.78
Δy	big	0.43	37.40	0.65
AΔlgρ	Default	0.45	41.10	0.41
HML	Default	0.46	42.40	0.15
SMB	big	0.48	46.60	0.08
Δprop	big	0.49	48.30	0.12
Δlgα	Dividend	0.50	50.10	0.18
R_m	Term	0.50	50.70	0.09

Next we compare all possible two-factor conditional models. Since we consider eight factors, we have total 28 possible two-factor combinations. For each two-factor combination, we try different conditioning variables for each factor, and choose the best model that has smallest HJ-distance. Table 5 reports the performance of factor combination (Δc, Δlogα) conditioning on different conditioning variables. Each factor has eight possible conditioning variables, so we get 64 possible specifications. The results are ranked by HJ-distance. The top specifications all have *big* as conditioning variable for Δc. The asymptotic distribution of T·J is weighted sum of 18 i.i.d. random variables with $\chi^2(1)$ distribution. Those top specifications can not be rejected, that is, pricing errors are not significantly different from zero.

TABLE 5. Compare Conditioning Variables for Two Factors

Factor 1	Conditioning Variable 1	Factor 2	Conditioning Variable 2	HJ-dist	T·J	p-value
Δc	big	Δlgα	Dividend	0.28	16.20	0.99
Δc	big	Δlgα	myfa	0.33	21.20	0.96
Δc	big	Δlgα	α	0.33	21.30	0.95
Δc	big	Δlgα	cay	0.33	21.40	0.96
...			…			
Δc	cay	Δlgα	α	0.55	59.50	0.00
Δc	Dividend	Δlgα	cay	0.55	59.90	0.00
Δc	α	Δlgα	cay	0.55	60.20	0.00
Δc	cay	Δlgα	cay	0.55	60.20	0.00

We repeat the same procedure, and choose the best conditioning variables for each two-factor combination. Results are reported in Table 6. The results are also ranked by HJ-distance. Those factors have good performance in one factor models still have good performance in two factor models. The factor Δc conditioning on *big* is still the most important factor. We also compare all possible three-factor conditional models. As we expected, the top specifications all have the factor Δc conditioning on *big* and adding the third factor does not improve the performance of two-factor models much. We therefore do not report the results of three-factor models.

The purpose of conditioning variable is to incorporate the time variation of risk premium. In conditional CCAPM, an asset's systematic risk is determined by correlation of its return with consumption growth conditioning on a state variable that reflects time variation in risk premium. Business income growth (*big*) is such a state variable. Heaton and Lucas (2000) find that household with high and variable business income hold less wealth in stocks than similarly wealthy households. Since they have more undiversifiable background risk, they are more risk averse against other source of risks, and then they invest less in stocks and demand higher compensation for holding stocks. Consequently, when the aggregate proprietary business income increases, expected risk premium goes up.

TABLE 6. Two-Factor Models with the Best Conditioning Variables

Factor 1	Conditioning Variable 1	Factor 2	Conditioning Variable 2	HJ-dist	T·J	p-value
Δc	big	Δlgα	Dividend	0.28	16.20	0.99
Δc	big	HML	Default	0.33	21.30	0.95
Δc	big	Δy	big	0.35	24.40	0.89
SMB	big	Δy	big	0.35	25.00	0.85
...						
HML	Default	Δlgα	Dividend	0.43	37.40	0.17
Δprop	big	AΔlgρ	Default	0.44	38.20	0.39
HML	Default	AΔlgρ	Default	0.44	39.10	0.25
R_m	Term	SMB	big	0.46	41.60	0.08

5. Conclusion

We run a horse race among eight proposed factors and eight conditioning variables, using same data sets and same method to compare their performances in explaining the cross-section of stock returns. We compare all possible linear factor model specifications that arise by combining these factors and conditioning variables. Factor specific conditioning variables are used to reduce the number of parameters that need to be estimated. We find that the consumption risk factor (Δc) with lagged business income growth (*big*) as the conditioning variable performs the best.

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